

CHAPMAN & HALL/CRC FINANCIAL MATHEMATICS SERIES

An Introduction to
**CREDIT RISK
MODELING**

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CHAPMAN & HALL/CRC

A CRC Press Company

Boca Raton London New York Washington, D.C.

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Acknowledgements

Christian Bluhm would like to thank his wife Tabea and his children Sarah and Noa for their patience during the writing of the manuscript. Without the support of his great family this project would not had come to an end. Ludger Overbeck is grateful to his wife Bettina and his children Leonard, Daniel and Clara for their ongoing support.

We very much appreciated feedback, support, and comments on the manuscript by our colleagues.

Questions and remarks of the audiences of several conferences, seminars and lectures, where parts of the material contained in this book have been presented, in many ways improved the manuscript. We always enjoyed the good discussions on credit risk modeling issues with colleagues from other financial institutions. To the many people discussing and sharing with us their insights, views, and opinions, we are most grateful.

Disclaimer

This book reflects the personal view of the authors and not the opinion of HypoVereinsbank, Deutsche Bank, or Allianz. The contents of the book has been written for educational purposes and is neither an offering for business nor an instruction for implementing a bank-internal credit risk model. The authors are not liable for any damage arising from any application of the theory presented in this book.

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