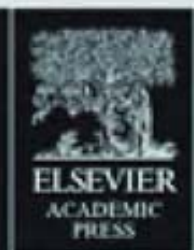


ACADEMIC PRESS ADVANCED FINANCE SERIES



# Quantitative Finance for **Physicists**

AN INTRODUCTION



ANATOLY B. SCHMIDT

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**QUANTITATIVE FINANCE  
FOR PHYSICISTS:  
AN INTRODUCTION**

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ANATOLY B. SCHMIDT



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