

WELCOME TO THE RANDOM WORLD OF HAPPY HARRY—
famed restaurateur, happy hour host, community figure, former
semi-pro basketball player, occasional software engineer, talent agent, bud-
ding television star, world traveller, nemesis of the street gang called the
Mutant Creepazoids, theatre patron, supporter of precise and elegant use
of the English language, supporter of the war on drugs, unsung hero of the
fairy tale *Sleeping Beauty*, and the target of a vendetta by the local chap-
ter of the Young Republicans. Harry and his restaurant are well known
around his Optima Street neighborhood both to the lovers of fine food and
the public health service. Obviously this is a man of many talents and
experiences who deserves to have a book written about his life.



Sidney Resnick

Adventures in Stochastic Processes

with Illustrations



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Preface

While this is a book about Harry and his adventurous life, it is primarily a serious text about stochastic processes. It features the basic stochastic processes that are necessary ingredients for building models of a wide variety of phenomena exhibiting time varying randomness.

The book is intended as a first year graduate text for courses usually called *Stochastic Processes* (perhaps amended by the words “Applied” or “Introduction to . . .”) or *Applied Probability*, or sometimes *Stochastic Modelling*. It is meant to be very accessible to beginners, and at the same time, to serve those who come to the course with strong backgrounds. This flexibility also permits the instructor to push the sophistication level up or down. For the novice, discussions and motivation are given carefully and in great detail. In some sections beginners are advised to skip certain developments, while in others, they can read the words and skip the symbols in order to get the content without more technical detail than they are ready to assimilate. In fact, with the numerous readings and variety of problems, it is easy to carve a path so that the book challenges more advanced students, but remains instructive and manageable for beginners. Some sections are starred and come with a warning that they contain material which is more mathematically demanding. Several discussions have been modularized to facilitate flexible adaptation to the needs of students with differing backgrounds. The text makes crystal clear distinctions between the following: proofs, partial proofs, motivations, plausibility arguments and good old fashioned hand-waving.

Where did Harry, Zeke and the rest of the gang come from? Courses in Stochastic Processes tend to contain overstuffed curricula. It is, therefore, useful to have quick illustrations of how the theory leads to techniques for calculating numbers. With the Harry vignettes, the student can get in and out of numerical illustrations quickly. Of course, the vignettes are not meant to replace often stimulating but time consuming real applications. A variety of examples with applied appeal are sprinkled throughout the exposition and exercises. Our students are quite fond of Harry and enjoy psychoanalyzing him, debating whether he is “a polyester sort of guy” or the “jeans and running shoes type.” They seem to have no trouble discerning the didactic intent of the Harry stories and accept the need for some easy numerical problems before graduating to more serious ones. Student culture has become so ubiquitous that foreign students who are